ELECTROMAGNETIC WAVES

8.1 Introduction: We have discussed and learnt that an electric current produces a magnetic field around itself while flowing through a conductor. We have also learnt that a changing magnetic field (*w.r.t.* the time) produces an electric field. The question arises that, ***can a changing electric field (w.r.t. the time) produce a magnetic field***?

The great Scientist *James Clerk Maxwell* (1831-1879), claimed that this is the fact: an electric field changing with time produces a magnetic field. He noticed that the *Ampere's Circuital Law* is inconsistent as it makes non-unique predictions for the magnetic field in some situations, where electric current (and hence electric field) changes *w.r.t.* the time. He proved that the consistency requires an additional source of magnetic field, in such cases, and ***the corresponding additional source of magnetic field*** was named as the ***displacement current***. With the help of this new discovery of the displacement current, the laws of electricity and magnetism became symmetrical *w.r.t.* each other.

James Clerk Maxwell formulated a set of four equations, well known as ***Maxwell's Equations***. He predicted with the help of these four equations that the "*time and space dependent electric field and magnetic field propagate mutually perpendicular to each other as transverse waves, and are known as* ***electromagnetic waves***". He discovered that the electromagnetic waves travel with the speed of light and came to an unbeatable unique conclusion that light is an electromagnetic wave.

Heinrich Hertz, in 1887, successfully demonstrated the existence of electromagnetic waves with the help of artificial sources of electromagnetic waves. A few years later, *Guglielmo Marconi* (*Italy*) succeeded in transmitting artificially produced electromagnetic waves over distances of several kilometers through the air. This achievement brought a revolution in electronic communication, which we are witnessing today in the use of the gadgets of our daily life.

We will learn electromagnetic waves, Maxwell's Equations, the transverse nature of electromagnetic waves, performance parameters (like: *energy density, intensity, momentum of electromagnetic waves etc.*), the sources for their origination, their properties, electromagnetic spectrum, effect of earth's atmosphere on electromagnetic radiation in this chapter in detail.

8.2 Maxwell's Displacement Current: Maxwell noticed the inconsistency in the *Ampere's Circuital Law*, and gave a correction to it in the form of *displacement current*.

Inconsistency in the Ampere's Circuital law: Ampere's Circuital Law states that "*the line integral of the magnetic field ($\vec{B}$) along a closed path is proportional to the total current* (ΣI) *passing through that closed path*".

i.e. $$\oint_l \vec{B}.d\vec{l} = \mu_0 \Sigma I \tag{8.1}$$

Maxwell observed in 1864 that above equation is logically inconsistent. He showed this inconsistency with the help of a setup for charging of a capacitor. Consider the setup shown in the Fig. 8.1(a), for charging of a capacitor plates. The current I flows through the connecting wires in the circuit, which changes with time. This current produces an electric field inside the capacitor by charging the capacitor plates. Consider two planer loops l_1 and l_2, as shown in the Fig. 8.1 (a), the loop l_1 is just at the left of the capacitor, while the loop l_2 is in between the capacitor plates, with their planes parallel to the capacitor plates.

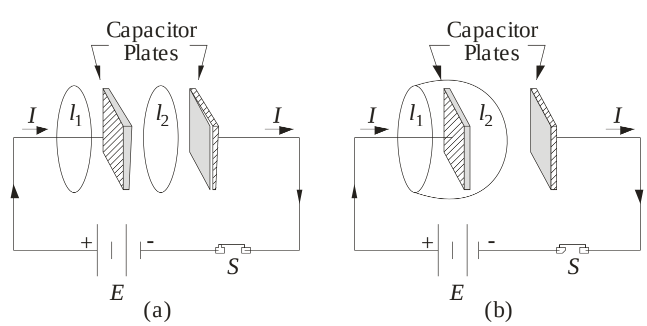


Fig. 8.1

Observe the loop l_1, a current I is flowing through this closed path (l_1) in the connecting lead. So, we may write down according to *Ampere's Circuital Law*:

$$\oint_{l_1} \vec{B}.d\vec{l} = \mu_0 I \tag{8.2}$$

Now, observe the loop l_2 between the two plates of the capacitor, no current is flowing through this closed path (l_2). So, we may write down according to *Ampere's Circuital Law*:

$$\oint_{l_2} \vec{B}.d\vec{l} = 0 \tag{8.3}$$

If the loops l_1 and l_2 are in very close proximity of the left capacitor plate (which in fact is), then we must have:

$$\oint_{l_1} \vec{B}.d\vec{l} = \oint_{l_2} \vec{B}.d\vec{l} \tag{8.4.1}$$

Which is not true, the reader may observe it from the inspection of equation (8.2) and (8.3) that actually:

$$\oint_{l_1} \vec{B}.d\vec{l} \neq \oint_{l_2} \vec{B}.d\vec{l} \tag{8.4.2}$$

This is the inconsistency of the *Ampere's Circuital Law*, which was modified by *Maxwell* introducing the concept of the *displacement current*.

Maxwell's Modification of Ampere's Circuital Law: Maxwell followed the symmetry consideration between resultant electric field due to changing magnetic field and resultant magnetic field due to changing electric field to modify *Ampere's circuital law*.

Consider the Faraday's law of electromagnetic induction, *i.e.* an induced emf is produced in a coil due to the change in magnetic field associated with it, so it indicates that an electric field is being produced by a change in magnetic field. Maxwell insisted that if this law holds good, then it must also be true that a magnetic field will be produced due to a change in the electric field, associated with a changing current and / or associated with a changing electric field. Maxwell called this changing current as *displacement current* (I_d), to distinguish it from the usual conduction current caused by the drift of the electrons.

So, "*the displacement current may be defined as a current which exists in addition to the normal conduction current whenever the electric field and hence the electric flux changes with time*".

He represented the displacement current, to maintain the dimensional consistency, in the form:

$$I_d = \varepsilon_0 \frac{d\,\phi_E}{d\,t} \tag{8.5}$$

Where, ϕ_E = Electric Field × Area = $\left|\vec{E}\right| \times A$, *i.e.* the electric flux across the considered loop.

The displacement current (I_d) may be a changing conduction current flowing through a conductor or may be associated with an equivalent effect of a changing electric field.

So, the total current across the loop may now be given as:

$$I = I_C + I_d = I_C + \varepsilon_0 \frac{d\,\phi_E}{d\,t} \tag{8.6}$$

So, the modified form of the *Ampere's circuital law* but proposed by the *Maxwell* may be given as:

$$\oint_{l} \vec{B}.d\vec{l} = \mu_0 \Sigma\left[I_C + \varepsilon_0 \frac{d\,\phi_E}{d\,t}\right] \tag{8.7}$$

Consistency of Modified Ampere's Law: Observe the loop l_1, a current I is flowing through this closed path (l_1) in the connecting lead and there is no electric flux ($\phi_E = 0$). So, we may write down according to *Modified Ampere's Law*:

$$\oint_{l_1} \vec{B}.d\vec{l} = \mu_0 I \tag{8.8}$$

Now, observe the loop l_2 between the two plates of the capacitor, there is no conduction current ($I_c = 0$) flowing through this closed path (l_2) but displacement current is there ($I_d \neq 0$), associated with a changing electric field between the capacitor plates. So, we may write down according to *Modified Ampere's Law*:

$$\oint_{l_2} \vec{B}.d\vec{l} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} \tag{8.9}$$

If area of the capacitor plates is A, and the charge on the capacitor plates at any instant of time (t) during the process of charging is q, the electric field in the gap between the capacitor plates may be given as:

$$\left|\vec{E}\right| = \frac{q}{\varepsilon_0 A} \tag{8.10}$$

or, $$\left|\vec{E}\right| A = \frac{q}{\varepsilon_0}$$

So, the electric flux, $\phi_E = \left|\vec{E}\right| A = \dfrac{q}{\varepsilon_0}$ (8.11)

Putting the value of electric flux (ϕ_E) in the equation (8.9):

$$\oint_{l_2} \vec{B}.d\vec{l} = \mu_0 \varepsilon_0 \frac{d}{dt}\left(\frac{q}{\varepsilon_0}\right) = \mu_0 \frac{dq}{dt} = \mu_0 I \tag{8.12}$$

Now, $$\oint_{l_1} \vec{B}.d\vec{l} = \oint_{l_2} \vec{B}.d\vec{l} = \mu_0 I \tag{8.13}$$

This is a consistent result and proves the consistency of the *Modified Ampere's Law.*

Continuity of current: After modification in the *Ampere's Circuital Law*, the property of continuity of current is again established that the sum ($I_C + I_d$) of the conduction current and the displacement current remains continuous and constant throughout the circuit, even when the individual currents I_c and I_d may not be continuous. For example consider the setup shown in the Fig. 8.1, the current in the conducting leads is conduction current (I_c) and the displacement current is absent ($I_d = 0$).

So, $I_C = I$ and, $I_d = 0$ (8.14)

While the current between the capacitor plates is displacement current (I_d) and the conduction current is absent ($I_C = 0$).

$$I_C = 0 \quad \text{and,} \quad I_d = \varepsilon_0 \frac{d}{dt}\left(\frac{q}{\varepsilon_0}\right) = \frac{dq}{dt} = I \tag{8.15}$$

So, the displacement current satisfies the condition of continuity of the current.

8.3 Induced Magnetic Field due to the Displacement Current: The displacement current (I_d) also produces the same physical effects as that of the conduction current (I_C). So, the displacement current is also associated with a magnetic field as similar to that of the conduction current. Consider the example of charging of a parallel plate capacitor, the charging current flows in the conducting leads in the form of conduction current (I_C) and the charging current exists in the form of the displacement current (I_d) between the plates of the parallel plate capacitor. So, the displacement current (I_d) in between the parallel plates of the capacitor also produces a magnetic field as similar to that of the conduction current producing outside the parallel plates of the capacitor, *i.e.* in a clock wise direction, if we are facing the plate *P* of the parallel plate capacitor from the left side as shown in the Fig. 8.2 (a). The direction of magnetic field *w.r.t.* the direction of the electric field is further elaborated in the Fig. 8.2 (b), if we are facing the plate *P* of the parallel plate capacitor from the left side. This shows that the direction of magnetic field produced is always perpendicular to the changing electric field due to which it is being produced.

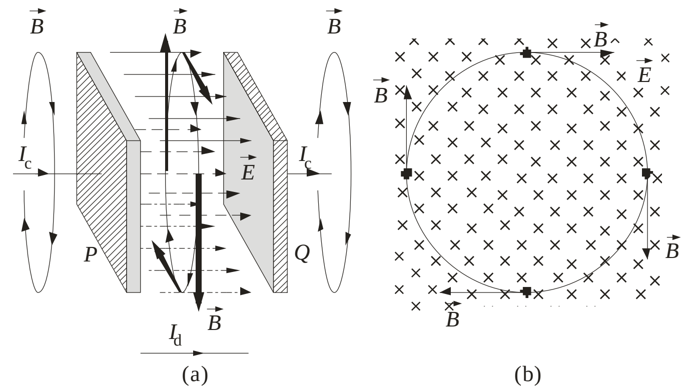


Fig. 8.2

Consequences of Displacement Current: The concept of displacement current made the laws of electricity and that of the laws of magnetism completely symmetrical to each other. The electric field may be produced with the help of a changing magnetic field (*Faraday's Law of Electromagnetic Induction*), while the magnetic field may be produced with the help of a changing electric field (*Modified Ampere's Law* with the help of *Maxwell's correction*).

Another very important and the most significant consequence of the displacement current is the mutual existence and perpendicular direction of the electric field and the magnetic field (*known as electromagnetic field*) *associated with each other, whenever there is a change in any of them*. ***This consequence explained the existence and the methods to produce the artificial electromagnetic waves for present electronic wireless communications***.

Important Properties of Displacement Current: The important properties of the displacement current are enlisted below:

i) The displacement current (I_d) exists only when there is a change in electric flux, so it is a transient current and unlike the conduction current it may not exist under steady state conditions.

ii) It is not an actual current, which is physically flowing somewhere in the circuit. It only adds to the current density in *Ampere's circuital Law*. This current is producing a magnetic field, so is called as the displacement current (I_d).

iii) The magnitude of the displacement current (I_d) is proportional to the rate of transfer of the charge from one plate of the capacitor to another plate of the capacitor.

iv) The displacement current (I_d) along with the conduction current (I_C) satisfies and explains the continuity of the current across a circuit.

Problem 8.1: *A parallel plate capacitor, having circular plates of radius* 5 *cm, is being charged so that the electric field between the plates of the capacitor rises steadily at the rate of* 10^{12} *V/m-sec. Determine the value of the displacement current between the parallel plates of the capacitor.*

Solution: $r = 5$ cm, $\dfrac{d|\vec{E}|}{dt} = 10^{12}$ V/m-sec

The displacement current between the parallel plate capacitor, where the electric field is changing continuously, may be given as:

$$I_d = \varepsilon_0 \frac{d\,\phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(|\vec{E}|\,A\right) = \varepsilon_0 A \frac{d|\vec{E}|}{dt} = \varepsilon_0 (\pi r^2) \frac{d|\vec{E}|}{dt}$$

$$= 8.85 \times 10^{-12} \times \pi \times (0.05)^2 \times 10^{12} = 0.0695 \text{ A} = 69.5 \text{ mA}$$

Problem 8.2: *The voltage between the plates of a parallel plate capacitor of capacitance* 1 μ*F is changing at the rate of* 5 *V/sec. Determine the value of the displacement current between the parallel plates of the capacitor.*

Solution: $C = 1\ \mu\text{F}$, $dV / dt = 5$ V/sec

The displacement current between the parallel plate capacitor, where the voltage and hence the electric field is changing continuously, may be given as:

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d(|\vec{E}|A)}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{VA}{d}\right) = \frac{\varepsilon_0 A}{d} \times \frac{dV}{dt} = C \times \frac{dV}{dt}$$

$$= 1 \times 10^{-6} \times 5 = 5 \times 10^{-6}\ \text{A} = 5\ \mu\text{A}$$

Problem 8.3: *A parallel plate capacitor, of plate area* 50 *cm*^{2} *and a plate separation of* 3 *mm, is charged initially to* 80 μ*C. The dielectric medium between the plates of the capacitor becomes slightly conducting due to a nearby radio-active source and the capacitor losses the charge at a rate of* 1.5×10^{-8} *C/sec. Determine the magnitude and the direction of the displacement current. Also, determine the value of magnetic field induced between the plates.*

Solution: $A = 50\ \text{cm}^2$, $d = 3$ mm, $q = 80\ \mu\text{C}$, $\frac{dq}{dt} = 1.5 \times 10^{-8}$ C/sec

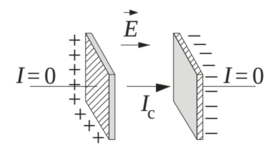


Fig. 8.3

The situation of the leakage of charge (and hence the flow of a conduction current) between the parallel plates of a capacitor through its dielectric medium is shown in the Fig. 8.3.So, a conduction current (I_c) will flow from the positively charged plate of the capacitor towards the negatively charged plate of the capacitor. According to the continuity of the current the sum of conduction current and the displacement current remains constant in the circuit, which is zero in this case as no current is flowing outside the capacitor.

So, $I_C + I_d = 0$ or, $I_d = -I_C$ (8.16)

The displacement current between the parallel plates of the capacitor, where the charge is leaking continuously, may be given as:

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d(|\vec{E}|A)}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{q}{\varepsilon_0 A} \times A\right) = \frac{dq}{dt} = 1.5 \times 10^{-8}\ \text{A}$$

Using the Modified Ampere's Law, we may write the expression:

$$\oint_l \vec{B}.d\vec{l} = \mu_0 (I_C + I_d) = \mu_0 \times 0 = 0 \qquad \text{So,} \qquad \vec{B} = 0$$

Problem 8.4: *A parallel plate capacitor, having circular plates of radius* 12 *cm, is being charged in a circuit drawing a current of* 0.15 *A from the source. Determine: i*) *the magnetic field at the axis of the capacitor, ii*) *at a distance of* 6.5 *cm from the axis of the capacitor, iii*) *at a distance of* 15 *cm from the axis of the capacitor, iv*) *the distance from the axis of the capacitor at which the magnetic field is maximum.*

Solution: $R = 12$ cm, $I_{C(Conducting\ leads)} = 0.15$ A, $r_1 = 0$ cm, $r_2 = 6.5$ cm, , $r_3 = 15$ cm

The displacement current inside the capacitor may be given by the relationship of current continuity:

$$I_{C(capacitor)} + I_{d(capacitor)} = I_{C(conducting\ leads)} + I_{d(conducting\ leads)}$$

or, $0 + I_{d(capacitor)} = 0.15 + 0$ or, $I_{d(capacitor)} = 0.15$ A

The current density (displacement current) inside the capacitor may be given as: $J_d = \dfrac{I_d}{\pi R^2}$

Now consider the Fig. 8.4. The dark circle with radius (R) is showing the capacitor plates, and the inner dotted circle is having a radius (r) which is smaller than the radius of capacitor plates. If the direction of current is normally into the plane of the paper and away from the reader, the corresponding direction of induced magnetic field is shown in the figure.

The magnetic field due to the current within the circle having a radius ***r* < *R*** may be given, according to *Modified Ampere's Law*, as:

$$\int_0^r \vec{B}.d\vec{l} = \mu_0 I_{d(r)} = \mu_0 \times (J_d \times A_r) = \mu_0 \times \frac{I_d}{\pi R^2} \times \pi r^2$$

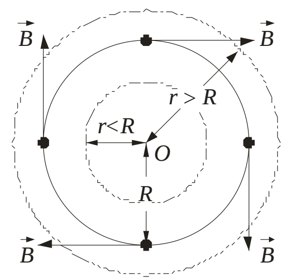


Fig. 8.4

or, $\quad B \times 2\pi r = \dfrac{\mu_0 I_d r^2}{R^2} \quad$ or, $\quad B = \dfrac{\mu_0 I_d r}{2\pi R^2}$

So, the magnetic field at the axis of the capacitor (r = 0) may be given as:

$$B_{axis} = \frac{4\pi \times 10^{-7} \times 0.15 \times 0}{2\pi \times (12 \times 10^{-2})^2} = 0 \text{ Tesla}$$

And, the magnetic field at a distance r = 6.5 cm from the axis of the capacitor may be given as:

$$B_{6.5\text{ cm}} = \frac{4\pi \times 10^{-7} \times 0.15 \times 6.5 \times 10^{-2}}{2\pi \times (12 \times 10^{-2})^2} = 1.354 \times 10^{-7} \text{ T}$$

Now again reconsider the Fig. 8.4. The dark circle with radius (R) is showing the capacitor plates, and the outer dotted circle is having a radius (r) which is greater than the radius of capacitor plates.

The magnetic field due to the displacement current within the circle having a radius ***r* > *R*** may be given, according to *Modified Ampere's Law*, as:

$$\int_0^r \vec{B}.d\vec{l} = \mu_0 I_{d(r)} = \mu_0 \times (J_d \times A_r) = \mu_0 \times \frac{I_d}{\pi R^2} \times \pi R^2$$

As, the displacement current is flowing between the plates of the capacitor within the circumference of the capacitor plates only and not outside the capacitor plates away from the dark circle ($r > R$).

or, $\quad B \times 2\pi r = \mu_0 I_d \quad$ or, $\quad B = \dfrac{\mu_0 I_d}{2\pi r}$

So, the magnetic field at a distance r = 15 cm from the axis of the capacitor may be given as:

$$B_{15\text{ cm}} = \frac{4\pi \times 10^{-7} \times 0.15}{2\pi \times (15 \times 10^{-2})} = 2 \times 10^{-7} \text{ T}$$

The reader may conclude that the maximum magnetic field exists at the point $r = R$ = 12 cm, and may be given as:

$$B_{12\text{ cm}} = \frac{4\pi \times 10^{-7} \times 0.15}{2\pi \times (12 \times 10^{-2})} = 2.5 \times 10^{-7} \text{ T}$$

Problem 8.5: *A parallel plate capacitor, having circular plates of radius* 1 *m and a capacitance of* 1 *nF, is connected for charging in series with a resistance R* = 1 *MΩ across a* 2 *V battery, at t* = 0 *sec. Determine the magnetic field at a point P, in between the parallel plates of the capacitor and half way between the center and the circumference of the plates after* 10^{-3} *sec.* [NCERT]

Solution: $R = 1$ m, $C = 1$ nF, $R = 1$ MΩ, $V = 2$ V, $r = 0.5$ m, $t = 10^{-3}$ sec

The expression for the rising conduction current in an *R-C* series circuit may be given as:

$$i_C = \frac{E}{R} \times e^{-(t/RC)}$$

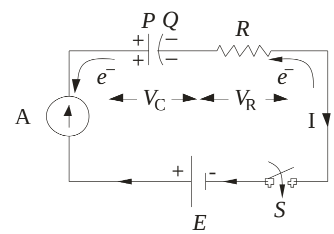


Fig. 8.5

So, the displacement current inside the capacitor may be given by the relationship of current continuity:

$$I_{C(capacitor)} + I_{d(capacitor)} = I_{C(conducting\ leads)} + I_{d(conducting\ leads)}$$

or, $$0 + I_{d(capacitor)} = i_C + 0$$

or, $$I_{d(capacitor)} = i_C = \frac{E}{R} \times e^{-(t/RC)}$$

The current density (displacement current) inside the capacitor may be given as: $J_d = \dfrac{I_d}{\pi R^2}$

The magnetic field due to the current within the circle having a radius ***r*** < ***R*** may be given, according to *Modified Ampere's Law*, as:

$$\int_0^r \vec{B}.d\vec{l} = \mu_0 I_{d(r)} = \mu_0 \times (J_d \times A_r) = \mu_0 \times \frac{I_d}{\pi R^2} \times \pi r^2$$

or, $$B \times 2\pi r = \frac{\mu_0 I_d r^2}{R^2}$$ or, $$B = \frac{\mu_0 I_d r}{2\pi R^2}$$

So, the magnetic field at a distance $r = 0.5$ m at the time instant $t = 10^{-3}$ sec may be given as:

$$B_{(0.5\ m,\ 10^{-3}\ \text{sec})} = \frac{\mu_0 I_d r}{2\pi R^2} = \frac{\mu_0 i_c r}{2\pi R^2} = \frac{\mu_0 r}{2\pi R^2} \times \frac{E}{R} \times e^{-(t/RC)}$$

$$= \frac{4\pi \times 10^{-7} \times 0.5}{2\pi \times (1)^2} \times \frac{2}{1\times 10^6} \times e^{-(10^{-3}/1\times 10^6 \times 1\times 10^{-9})} = 7.358 \times 10^{-14}\ \text{T}$$

Problem 8.6: *How would a displacement current of* 2 *A be established between the two parallel plates of a capacitor of* 1 μ*F.*

Solution: $I_d = 2$ A, $C = 1$ μF

The displacement current across the parallel plates of a capacitor may be given as:

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d(|\vec{E}| A)}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{VA}{d}\right) = \frac{\varepsilon_0 A}{d} \times \frac{dV}{dt} = C \times \frac{dV}{dt}$$

or, $$\frac{dV}{dt} = \frac{I_d}{C} = \frac{2}{1\times 10^{-6}} = 2 \times 10^6 \text{ V/sec} = 2 \text{ MV/sec}$$

So, this much displacement current may be setup by changing the potential difference at a rate of 2 MV/sec.

Problem 8.7: *An air filled capacitor, having two circular parallel plates of radius* 10 *cm and a plate separation of* 2 *mm, is charged by an external battery. The charging current is constant and is equal to* 0.5 *A. Determine: i*) *the capacitance of the capacitor, ii*) *the rate of change of potential difference across the plates of the capacitor, iii*) *the displacement current inside the capacitor.*

Solution: R = 10 cm, d = 2 mm, I_C = 0.5 A

The capacitance of the capacitor may be given as:

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times\pi\times(0.1)^2}{2\times10^{-3}} = 1.3902 \times 10^{-10}\text{ F} = 139.02\text{ pF}$$

The displacement current inside the capacitor may be given by the relationship of current continuity:

$$I_{C(capacitor)} + I_{d(capacitor)} = I_{C(conducting\ leads)} + I_{d(conducting\ leads)}$$

or, $0 + I_{d(capacitor)} = I_C + 0$ or, $I_{d(capacitor)} = I_C = 0.5$ A

The displacement current across the parallel plates of a capacitor may also be given as:

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d(|\vec{E}|A)}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{VA}{d}\right) = \frac{\varepsilon_0 A}{d} \times \frac{dV}{dt} = C \times \frac{dV}{dt}$$

or, $$\frac{dV}{dt} = \frac{I_d}{C} = \frac{0.5}{139.01\times10^{-12}} = 3.5968 \times 10^{9}\text{ V/sec} = 3596.8\text{ MV/sec}$$

Problem 8.8: *An air filled parallel plate capacitor, having two plates of size* 30 *cm* × 15 *cm and a plate separation of* 2 *mm, is charged by an external battery and the charging current is found to be constant and is equal to* 100 *mA. Determine: i*) *the capacitance of the capacitor, ii*) *the rate of change of potential difference across the plates of the capacitor, iii*) *the displacement current inside the capacitor.*

Solution: A = 30 cm × 15 cm, d = 2 mm, I_C = 100 mA

The capacitance of the capacitor may be given as:

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times30\times15\times10^{-4}}{2\times10^{-3}} = 1.99125 \times 10^{-10}\text{ F} = 199.125\text{ pF}$$

The displacement current inside the capacitor may be given by the relationship of current continuity:

$$I_{C(capacitor)} + I_{d(capacitor)} = I_{C(conducting\ leads)} + I_{d(conducting\ leads)}$$

or, $0 + I_{d(capacitor)} = I_c + 0$ or, $I_{d(capacitor)} = I_C = 100$ mA

The displacement current across the parallel plates of a capacitor may also be given as:

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d(|\vec{E}|A)}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{VA}{d}\right) = \frac{\varepsilon_0 A}{d} \times \frac{dV}{dt} = C \times \frac{dV}{dt}$$

or, $$\frac{dV}{dt} = \frac{I_d}{C} = \frac{100\times10^{-3}}{199.125\times10^{-12}} = 5.02197 \times 10^{8}\text{ V/sec} = 502.197\text{ MV/sec}$$

Problem 8.9: *A parallel plate capacitor of capacitance C* = 0.1 μ*F is connected across an a.c. source of angular frequency* 500 *rad/sec. The value of conduction current is* 1 *mA. Determine the rms value of the voltage source. Also, determine the displacement current across the capacitor plates.*

Solution: $C = 0.1\ \mu F$, $\omega = 500$ rad/sec, $I_C = 1$ mA

The rms value of the voltage source may be given as:

$$V = I_C X_C = I_C \times \frac{1}{\omega C} = 1 \times 10^{-3} \times \frac{1}{500 \times 0.1 \times 10^{-6}} = 20\ \text{V}$$

The displacement current across the capacitor plates may be given by the relationship of current continuity:

$$I_{C(capacitor)} + I_{d(capacitor)} = I_{C(conducting\ leads)} + I_{d(conducting\ leads)}$$

or, $0 + I_{d(capacitor)} = I_C + 0$ or, $I_{d(capacitor)} = I_C = 1$ mA

Problem 8.10: *A parallel plate capacitor, having two circular plates each of radius* 10 *cm and a capacitance of* 200 *pF, is connected across a* 200 *V a.c. supply having an angular frequency of* 200 *rad/sec. Determine: i*) *the r.m.s. value of conduction current, ii*) *whether the displacement current is equal to the conduction current, iii*) *the peak value of displacement current, iv*) *the amplitude of magnetic field at a point* 2 *cm from the axis of the capacitor.*

Solution: $R = 10$ cm, $C = 200$ pF, $V = 200$ V, $\omega = 200$ rad/sec, $r = 2$ cm

The rms value of the conduction current may be given as:

$$I_C = \frac{V}{X_C} = \frac{V}{(1/\omega C)} = V \times \omega\, C = 200 \times 200 \times 200 \times 10^{-12} = 8 \times 10^{-6}\ \text{A} = 8\ \mu\text{A}$$

The displacement current may be given as:

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d}{dt}(|\vec{E}|A) = \varepsilon_0 A \times \frac{d}{dt}\left(\frac{V}{d}\right)$$

$$= \frac{\varepsilon_0 A}{d} \times \frac{dV}{dt} = C \times \frac{d}{dt}\left(\frac{q}{C}\right) = \frac{dq}{dt} = I_c$$

So, $I_d = I_C$ *i.e.* Displacement Current = Conduction Current

The peak value of displacement current may be given as:

$$(I_d)_0 = (I_C)_0 = \sqrt{2} \times I_c = \sqrt{2} \times 8 = 8\sqrt{2}\ \mu\text{A} = 11.314\ \mu\text{A}$$

The current density (displacement current) inside the capacitor may be given as: $J_d = \dfrac{I_d}{\pi R^2}$

The magnetic field due to the current within the circle having a radius ***r* < *R*** may be given, according to *Modified Ampere's Law*, as:

$$\int_0^r \vec{B}.d\vec{l} = \mu_0 I_{d(r)} = \mu_0 \times (J_d \times A_r) = \mu_0 \times \frac{I_d}{\pi R^2} \times \pi r^2$$

or, $B \times 2\pi r = \dfrac{\mu_0 I_d r^2}{R^2}$ or, $B = \dfrac{\mu_0 I_d r}{2\pi R^2}$

So, the amplitude of magnetic field at a distance $r = 2$ cm from the axis of the capacitor may be given as:

$$B_{(2\ \text{cm})} = \frac{\mu_0 (I_d)_0 r}{2\pi R^2} = \frac{4\pi \times 10^{-7} \times 8 \times \sqrt{2} \times 10^{-6} \times 0.02}{2\pi \times (0.10)^2} = 4.526 \times 10^{-12}\ \text{T}$$

8.4 Maxwell's Equation: Maxwell came to the conclusion that all the basic principles of electromagnetism can be formulated in terms of four equations only, collectively known as *Maxwell's equations*. All the four *Maxwell's equations* were given below assuming that there is no magnetic or dielectric material (*i.e.* $\mu_r = 1$ and $\varepsilon_r = 1$). If any magnetic or dielectric material is involved μ_0 will be replaced by $\mu_0 \mu_r$ and ε_0 will be replaced by $\varepsilon_0 \varepsilon_r$ in all the equations at respective places.

***i*) Gauss Law for Electrostatics:** This states that the electric flux through a closed surface (S) is $\frac{1}{\varepsilon_0}$ times the total charge (q) enclosed by that surface.

i.e. $$\oint_S \vec{E}.d\vec{S} = \frac{q}{\varepsilon_0} \qquad (8.16)$$

Important consequences of this law are: *i*) *total charge on an insulated conductor resides only at the outer surface of the conductor*, *ii*) *the electrostatic force between two charges is inversely proportional to the square of distance between them*.

***ii*) Gauss Law for Magnetism:** This states that the magnetic flux through any closed surface (S) is always zero.

i.e. $$\oint_S \vec{B}.d\vec{S} = 0 \qquad (8.17)$$

This law explains that the isolated magnetic poles (*monopoles*) *may not exist in nature*.

***iii*) Faraday's Law of Electromagnetic Induction:** This states that a changing magnetic field can induce an electric field inside an electrical conductor. *The induced emf in a closed circuit, due to changing magnetic field, is equal to the rate of change of magnetic flux linkages of the closed circuit*.

i.e. $$\oint_l \vec{E}.d\vec{l} = -\frac{d\lambda_B}{dt} = -N\frac{d\phi_B}{dt} \qquad (8.17)$$

***iv*) Modified Ampere's Law:** This states that *the line integral of the magnetic field around any closed path around the flow of a current is equal to* μ_0 *times the total current* (*the sum of the conduction and the displacement currents*).

i.e. $$\oint_l \vec{B}.d\vec{l} = \mu_0 [I_C + I_d] = \mu_0 \left[I_C + \varepsilon_0 \frac{d\phi_E}{dt} \right] \qquad (8.18)$$

This law also states that the displacement current, associated with a changing electric field, produces a magnetic field as similar to that of the conduction current.

8.5 Maxwell's Prediction of Electromagnetic Waves: Maxwell theoretically predicted the existence of electromagnetic waves in 1865. He reasoned the predication of existence of the electromagnetic waves as given below.

"*A time varying magnetic field is a source of changing electric field, according to* ***Faraday's Law of Electromagnetic Induction***".

On the basis of theoretical studies, we already have discussed, Maxwell argued that "*A time varying electric field must be a source of changing magnetic field, according to* ***Modified Ampere's Law***".

Above conclusion indicates that a change in either field (electric / magnetic field) will produce another field. We already have seen that both the mutually dependent fields (electric / magnetic field) are mutually perpendicular to each other. And both the mutually changing fields are having wave like properties, as their amplitudes are changing continuously oscillating between their positive maxima to negative maxima like a sinusoidal a.c. supply.

Maxwell, thus, developed an idea that a wave of electric and magnetic fields both varying with space and time must exist, one field providing the source to the other progressing in the same direction but having mutual perpendicular orientations. He named such a wave as ***electromagnetic wave***, which in deed was in existence naturally from a very-very long period in the form of light and other waves.

Mathematical Representation of Electromagnetic Wave: Consider the Fig. 8.6, in which an electromagnetic wave progressing along positive *X*-direction is shown. The electric field ($\vec{E}$) is oscillating along the *Y*-direction, while the magnetic field ($\vec{B}$) is oscillating along the *Z*-direction. So, both the electric field and magnetic field are perpendicular to each other, while progressing along the third (*Z*-axis) direction. One field is acting as the source for the other and vice-versa. The values of electric field ($\vec{E}$) and magnetic field ($\vec{B}$) depends only on the values of *x* and *t*. The electric field vector ($\vec{E}$) may be given as:

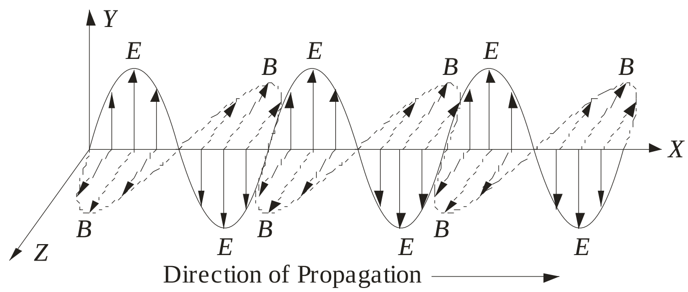


Fig. 8.6

$$\vec{E} = E_y\,\hat{j} = E_0 \sin(k\,x - \omega\,t)\,\hat{j} = E_0 \sin\left[2\pi\left(\frac{x}{\lambda} - \nu t\right)\right]\hat{j} = E_0 \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]\hat{j} \qquad (8.19)$$

Where, $k = (2\,\pi / \lambda)$ is propagation constant of the wave and the angular frequency is $\omega = 2\,\pi\,\nu$.

The reader may observe that; $E_x = E_z = 0$ (8.20)

Similarly, the magnetic field vector ($\vec{B}$) may be given as:

$$\vec{B} = B_z\,\hat{k} = B_0 \sin(k\,x - \omega\,t)\,\hat{k} = B_0 \sin\left[2\pi\left(\frac{x}{\lambda} - \nu t\right)\right]\hat{k} = B_0 \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]\hat{k} \qquad (8.21)$$

Where, $k = (2\,\pi / \lambda)$ is propagation constant of the wave and the angular frequency is $\omega = 2\,\pi\,\nu$.

The reader may observe that; $B_x = B_y = 0$ (8.22)

The amplitude of electric field vector ($\vec{E}$) is E_0 and that of the magnetic field vector ($\vec{B}$) is B_0.

Equation (8.19) for electric field vector and (8.21) for magnetic field vector shows that the variations in electric and magnetic fields are in same phase, *i.e.* both attain their maxima and minima simultaneously *w.r.t.* the time and phase (space location, *x*).

Also, the direction of propagation of the wave may be given as:

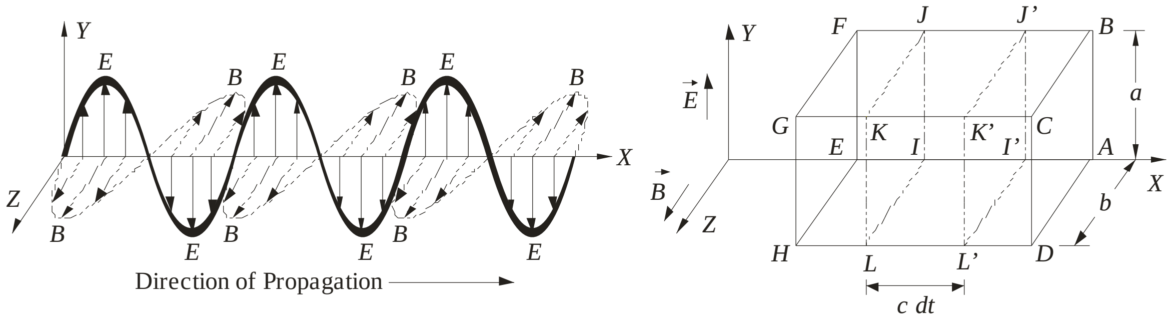
$$\text{Direction of propagation} = \vec{E} \times \vec{B} = \hat{j} \times \hat{k} = \hat{i} \qquad (8.23)$$

8.6 Speed of Electromagnetic Waves: An electromagnetic wave progressing in positive *X*-direction is shown in the Fig. 8.7 (a). Let the two positions of wave front at time instant *t* and ($t + dt$) is *IJKL* and *I'J'K'L'* as shown in the Fig. 8.7 (b). So, the distance travelled by the wave front in time *dt* may be given as:

$$LL' = c\,dt \qquad (8.24)$$

The height and width, of the parallelopiped assumed around the wave front, are *a* and *b* respectively. The electric field is crossing the face *ADHE* in perpendicular direction and the magnetic field is crossing the face *ABFE* in perpendicular direction.

Let us first consider the Magnetic Field crossing the Face *ABFE*:



(a) (b)

Fig. 8.7

The change in magnetic flux crossing the face *ABFE* in time *dt* may be given as:

$$d\phi_B = B \times \text{Area } JJ'I'I = B \times (a \times c\, dt)$$

or, $$\frac{d\,\phi_B}{d\,t} = B\,a\,c \tag{8.25}$$

The line integral of the electric field over the face *ABFE* may be given as (*remember that the electric field along the edge AB is zero, since wave front has not reached here till this time*):

$$\oint_{ABFE} \vec{E}.d\vec{l} = \int_A^B 0\; d\,l\cos 0^\circ + \int_B^F E\; d\,l\cos 90^\circ + \int_F^E E\; d\,l\cos 180^\circ + \int_E^A E\; d\,l\cos 90^\circ$$
$$= 0 + 0 - E\,a + 0$$

So, $$\oint_{ABFE} \vec{E}.d\vec{l} = -\,E\,a \tag{8.26}$$

According to *Faraday's Law of Electromagnetic Induction*, we have:

$$\oint_{ABFE} \vec{E}.d\vec{l} = -\frac{d\,\phi_B}{d\,t}$$

or, $-\,E\,a = -\,B\,a\,c$ or, $$\frac{E}{B} = c \tag{8.27}$$

Now consider the Electric Field crossing the Face *ADHE*:

The change in electric flux crossing the face *ADHE* in time *dt* may be given as:

$$d\phi_E = E \times \text{Area } LL'I'I = E \times (b \times c\, dt)$$

or, $$\frac{d\,\phi_E}{d\,t} = E\,b\,c \tag{8.28}$$

The line integral of the magnetic field over the face *ADHE* may be given as (*remember that the magnetic field along the edge AD is zero, since wave front has not reached here till this time*):

$$\oint_{ADHE} \vec{B}.d\vec{l} = \int_A^D 0\; d\,l\cos 0^\circ + \int_D^H B\; d\,l\cos 90^\circ + \int_H^E B\; d\,l\cos 180^\circ + \int_E^A B\; d\,l\cos 90^\circ$$
$$= 0 + 0 - B\,b + 0$$

So, $$\oint_{ADHE} \vec{B}.d\vec{l} = -\,B\,b \tag{8.29}$$

According to *Modified Ampere's Law*, we have:

$$\oint_{ADHE} \vec{B}.d\vec{l} = \mu_0 I_d = -\mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

or, $\quad -B\,b = -\mu_0 \varepsilon_0 E\,b\,c = -\mu_0 \varepsilon_0 (c\,B)\,b\,c$

or, $\quad c^2 = \dfrac{1}{\mu_0 \varepsilon_0}$

or, $\quad c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = \dfrac{1}{\sqrt{4\pi\times10^{-7}\times8.85\times10^{-12}}} = 2.999 \times 10^8$ m/sec

or, $\quad c = 3 \times 10^8$ m/sec (8.30)

This is exactly equal to the speed of light in vacuum. So, it was established that the electromagnetic waves travel with the speed of light. This fact lead Maxwell to predict that ***light is an electromagnetic wave***. *The emergence of the speed of light from purely electromagnetic considerations was the crowning achievement of Maxwell's electromagnetic theory*.

The speed of electromagnetic waves in any other medium / material (of relative permeability μ_r and relative permittivity ε_r) may be given as:

$$v = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \times \frac{1}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{n} \qquad (8.31)$$

Where, $n \rightarrow \sqrt{\mu_r \varepsilon_r}$ (refractive index of the material)

Since, the electric field and magnetic field are oscillating in mutually perpendicular directions and are propagating along the third perpendicular direction, so it was established that the ***electromagnetic waves are transverse in nature***.

8.7 Source of Electromagnetic Waves: A stationary charge produces only an electrostatic field, while a charge in uniform motion produces a magnetic field which does not change with time. So, neither the stationary charge nor a charge under uniform motion (steady currents) may produce an electromagnetic wave. Maxwell proposed that, "*an accelerating charge or in other words a continuously changing current w.r.t. the time may produce electromagnetic waves*".

A charge oscillating harmonically with time may serve this purpose, because a charge oscillating harmonically is under the influence of the continuous angular acceleration and may produce an oscillating electric field in its vicinity. This oscillating (continuously changing) electric field will give rise to an oscillating magnetic field in its vicinity. This may produce the electromagnetic waves, which travel along its propagation axis and the process continues since both the mutually perpendicular field acts as source for each other along the direction of propagation of the wave. So, *an electromagnetic wave originates from an oscillating charge*. The frequency of the electromagnetic wave so produced is equal to the frequency of oscillation of the charge. The electromagnetic waves receive the energy, to carry along with them, from the supply source in the oscillatory circuit.

We may conclude now, that in order to generate an electromagnetic wave of frequency ν, we need to setup an a.c. circuit having a current of same frequency ν. So, it is easier to generate low frequency electromagnetic waves, *e.g.* radio waves. However, it is not possible to demonstrate experimentally the origination of high frequency electromagnetic waves such as visible light. For example, the origination of yellow light requires an oscillator of frequency 6×10^{14} Hz, while the latest oscillators available are below the frequency range of 10^{11} Hz.

8.8 Hertz's Experiment: Maxwell predicted the electromagnetic waves theoretically in 1865. His great prediction had to wait for about 22 years for a German physicist, *Heinrich Hertz* for experimental verification of existence of electromagnetic waves.

Hertz used the oscillatory *L-C* circuit, shown in the Fig. 8.8, for producing the electromagnetic waves. The transmitter consists of two large square brass plates of sides 40 cm placed parallel to each other with a separation of 60 cm between them. The parallel brass plates are connected to two highly polished brass spheres (S_1 and S_2), with the help of thick wires, separated by a small distance of 2-3 cm. The two thick wires are connected to the secondary terminals of an induction coil.

Every time the current in the primary circuit of the induction coil is interrupted, a large potential difference is setup across the spheres S_1 and S_2 and the two parallel brass plates get charged. The high potential difference between the brass spheres (S_1 and S_2) ionizes the air between the spheres and the gap becomes conducting. The electrons and ions so produced oscillate back and forth across the gap S_1S_2. An oscillatory discharge of the plates occurs through the conducting air gap. The process of oscillatory discharge results in production of electromagnetic waves.

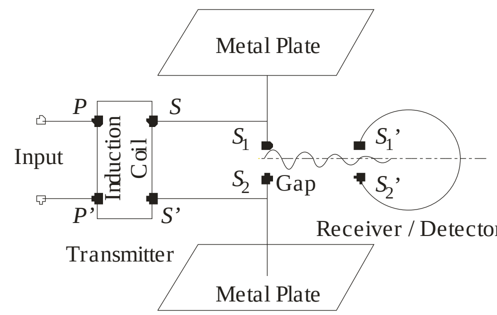


Fig. 8.8

The brass plates form a capacitor of low capacitance *C* (as the plate separation is too high) and connecting wires offers a low inductance *L* (being very thick wires of no turns). The oscillator *L-C* system generates electromagnetic waves of high frequency (ν) given by:

$$\nu = \frac{1}{2\pi\sqrt{LC}} \tag{8.32}$$

The receiver / detector consist of a closed circular stout wire, and its two terminals are terminating in two small polished brass spheres S_1' and S_2'. The electromagnetic waves on reaching the gap of the detector are associated with a sufficiently strong electric field which sets up a high potential difference across the gap $S_1'S_2'$, which causes tiny sparks jumping across the gap, proving the existence and propagation of electromagnetic waves from the transmitter to the receiver.

The electromagnetic signal produced by *Hertz* in the above experimentation was found to be having a frequency of 5×10^7 Hz. So, the wavelength of the electromagnetic signal produced by the *Hertz* may be given as:

$$\lambda = \frac{c}{\nu} = \frac{3\times10^8}{5\times10^7} = 6 \text{ m}$$

On the basis of above experimentation, *Hertz* demonstrated the various properties of electromagnetic waves as given below:

i) He observed that sparks attains a maximum value across the detector gap, when this gap is parallel to the gap of the transmitter. The sparks diminishes when the two gaps become perpendicular to each other. It indicates that the electric field transmitted from the transmitter is parallel to the two gaps, *i.e.* the direction of the electric field is perpendicular to the direction of the propagation of the electromagnetic wave. This demonstrates that *the electromagnetic waves are transverse in nature*.

ii) *Hertz* demonstrated the properties of reflection, refraction, diffraction and interference of the electromagnetic waves and established the fact beyond any doubt that *the electromagnetic radiations have the wave nature*.

iii) *Hertz* allowed the electromagnetic waves to fall on a large plane sheet of zinc. The reflected waves were superimposed on the incident waves, to produce stationary / standing electromagnetic waves. The wavelength (λ) of these standing waves was determined by measuring the distance between two consecutive nodes. The frequency of the waves was same as that of the source frequency (oscillator).

i.e. $$\nu = \frac{1}{2\pi\sqrt{LC}} \tag{8.33}$$

Hence, the speed of the electromagnetic waves was determined using the relationship $v = \nu\,\lambda$. It was observed that ***electromagnetic waves travel with the same speed as that of the light***.

iv) *Electromagnetic waves can be polarized*. We can test this fact with the help of a portable AM / FM radio provided with a telescopic antenna. It responds to the electric component of the electromagnetic signal from the broadcasting station. If the antenna is turned horizontally, the signal diminishes with increasing turning angle. The portable radio having horizontal antenna inside them are sensitive to the magnetic component of the electromagnetic signal. The signal is best received by these portable radios, when they are held horizontal.

8.9 History of the Observation of Electromagnetic Waves: The events of observation of electromagnetic waves are given below sequentially:

i) In 1865, *Maxwell* predicted the existence of electromagnetic waves with the help of purely theoretical considerations. He proved theoretically that *only an oscillating* (*accelerating*) *charge can originate electromagnetic waves*. Since, an oscillating charge is continuously oscillating, so it can continuously produce electromagnetic waves of same frequency as that of the oscillating charge.

ii) In 1887, *Hertz* gave the demonstration of experimental confirmation of existence of electromagnetic waves. He used an *L-C* oscillatory circuit for origination of electromagnetic waves. *He successfully produced the electromagnetic waves with a wavelength of 6 m and detected them after their propagation through air*.

iii) In 1895, *Sir J.C.Bose* successfully produced electromagnetic waves with much shorter wavelength (5 mm to 25 mm) during a public demonstration at Town Hall of Kolkata. *Bose* ignited gunpowder and rang a bell at a distance, using these shorter wavelengths (microwaves). *He produced these shorter electromagnetic waves and detected them after* 20 *m of their propagation through air*.

iv) In 1896, *Sir J.C.Bose* and *Guglielmo Marconi* discovered that if one of the terminals of the spark gap is connected to an antenna and the other terminal is grounded (earthed), then electromagnetic waves produced can be transmitted over a distance of several kilometers. They succeeded in transmitting electromagnetic waves across the British channel in 1899 and across the Atlantic ocean in 1901. Their experiments are a milestone in the evolution of radio communications now-a-days.

8.10 Energy Density of Electromagnetic Waves: The energy stored in an electric field (inside a capacitor) may be given as:

$$U_E = \frac{1}{2}\,C\,V^2 = \frac{1}{2} \times \frac{\varepsilon_0 A}{d} \times V^2 = \frac{1}{2} \times \varepsilon_0 A\,d \times \left(\frac{V}{d}\right)^2 = \frac{1}{2} \times \varepsilon_0 A\,d \times E^2 \tag{8.34}$$

So, the energy density of an electric field per unit volume, *i.e.* the energy stored inside an electric field of unit area and unit length may be given as:

$$u_E = \frac{1}{2} \times \varepsilon_0 A\,d \times E^2 = \frac{1}{2}\,\varepsilon_0 \times 1 \times 1 \times E^2 = \frac{1}{2}\,\varepsilon_0 E^2 \tag{8.35}$$

The energy stored in an inductor inside the magnetic field may be given as:

$$U_B = \frac{1}{2}\,L\,I^2 = \frac{1}{2} \times \frac{\mu_0 N^2 A}{l} \times \left(\frac{B}{\mu_0}\right)^2 = \frac{1}{2\mu_0} \times \frac{N^2 A}{l} \times B^2 \tag{8.36}$$

So, the energy density of a magnetic field per unit volume, *i.e.* the energy stored inside a magnetic field of unit area and unit length per turn may be given as:

$$u_B = \frac{1}{2\mu_0} \times \frac{(1)^2 \times 1}{1} \times B^2 = \frac{1}{2\mu_0} B^2 \qquad (8.37)$$

Now, the total energy density of the electromagnetic field of an electromagnetic wave may be given as:

$$u = u_E + u_B = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \qquad (8.38)$$

The electric field and the magnetic field vary in sinusoidal manner, so the effective energy density of the electromagnetic wave may be given as:

$$u_{rms} = \frac{1}{2} \varepsilon_0 E_{rms}^2 + \frac{1}{2\mu_0} B_{rms}^2 = \frac{1}{2} \varepsilon_0 \left(\frac{E_0}{\sqrt{2}}\right)^2 + \frac{1}{2\mu_0} \left(\frac{B_0}{\sqrt{2}}\right)^2$$

or, $$u_{rms} = \frac{1}{4} \varepsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2 \qquad (8.39)$$

We know from equation (8.27) that: $E = c\,B$

So, $$(u_E)_{rms} = \frac{1}{4} \varepsilon_0 E_0^2 = \frac{1}{4} \times \varepsilon_0 \times (c\,B_0)^2 = \frac{1}{4} \times \varepsilon_0 \times \frac{B_0^2}{\mu_0 \varepsilon_0} = \frac{1}{4\mu_0} B_0^2$$

or, $$(u_E)_{rms} = \frac{1}{4} \varepsilon_0 E_0^2 = \frac{1}{4\mu_0} B_0^2 = (u_B)_{rms} \qquad (8.40)$$

Hence, *the average energy density in the electric field is equal to the average energy density in the magnetic field in an electromagnetic wave.*

Putting equation (8.40) in (8.39):

$$(u)_{rms} = \frac{1}{4} \varepsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2 = \frac{1}{4} \varepsilon_0 E_0^2 + \frac{1}{4} \varepsilon_0 E_0^2 = \frac{1}{2} \varepsilon_0 E_0^2 = \varepsilon_0 E_{rms}^2 \qquad (8.41)$$

Also, $$(u)_{rms} = \frac{1}{4} \varepsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2 = \frac{1}{4\mu_0} B_0^2 + \frac{1}{4\mu_0} B_0^2 = \frac{1}{2\mu_0} B_0^2 = \frac{1}{\mu_0} B_{rms}^2 \qquad (8.42)$$

or, $$(u)_{rms} = \varepsilon_0 E_{rms}^2 = \frac{1}{\mu_0} B_{rms}^2 \qquad (8.43)$$

8.11 Intensity of Electromagnetic Wave: *The energy crossing per unit area per unit time in a direction parallel to the direction of propagation of the electromagnetic wave is known as the intensity of the electromagnetic wave.*

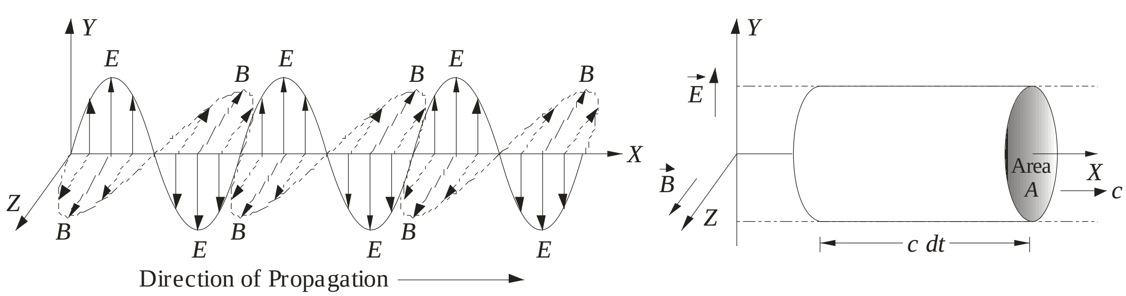


Fig. 8.9

Let us assume that an electromagnetic wave propagates along positive *X*-direction with a speed (speed of light *c*), as shown in the Fig. 8.9 (a). Consider a cylindrical volume with area of cross section *A* and length *c dt* along the *X*-axis around the electromagnetic wave, as shown in the Fig. 8.9 (b). The energy confined within this cylinder crosses the area *A* in time *dt* as the wave propagates with speed *c*. The energy confined within this cylinder may be given as:

$$U = \text{average energy density} \times \text{Volume} = u \times (c\,dt \times A) \qquad (8.44)$$

So, the intensity of the wave may be given as:

$$I = \frac{\text{Energy}}{\text{Area} \times \text{Time}} = \frac{U}{A \times dt} = \frac{u \times (c\,dt \times A)}{A \times dt} = u\,c$$

So, $$I = u\,c = \varepsilon_0\,E_{rms}^2\,c = \frac{1}{\mu_0}\,B_{rms}^2\,c \qquad (8.45)$$

We may conclude that, "*the intensity of the electromagnetic wave is proportional to the square of the electric field / magnetic field*".

Or, we may say that, "*the electric field / magnetic field of the electromagnetic wave is proportional to the square root of the intensity of the wave*".

8.12 Momentum of Electromagnetic Wave: An electromagnetic wave transports linear momentum, as it propagates through the space. If an electromagnetic wave is transporting the energy of U to a surface in time t, the total momentum delivered to the surface for complete energy absorption may be given as:

$$p_{\text{complete absorption}} = \frac{U}{c} \qquad (8.46)$$

If the wave is reflected back by the surface, the momentum delivered may be given as:

$$p_{\text{reflected back}} = \frac{2U}{c} \qquad (8.47)$$

As in the case of reflection, the momentum of the electromagnetic wave changes from p to $-p$.

8.13 Pressure Exerted by an Electromagnetic Wave: An electromagnetic wave exerts a pressure on the surface, when it falls on a surface. *This pressure exerted by the electromagnetic wave on a surface is known as radiation pressure*. The radiation pressure for an electromagnetic wave of intensity I may be given as:

$$P_r = \frac{I}{c} = \frac{u\,c}{c} = u \qquad (8.48)$$

In fact the tails of a comet always points away from the sun due to the radiation pressure of the light (electromagnetic wave emerging from the sun) on the comet.

8.14 Properties of Electromagnetic Waves: The peculiar properties of *E.M.* waves are enlisted below:

i) The *E.M.* waves are produced by the accelerating charges and do not require any material medium for their propagation.

ii) The direction of oscillation of electric field ($\vec{E}$) and that of the oscillation of magnetic field ($\vec{B}$) are mutually perpendicular to each other and the direction of propagation of the wave is along the third perpendicular direction. So, the *E.M.* waves are transverse in nature.

iii) The oscillations of electric field ($\vec{E}$) and that of magnetic field ($\vec{B}$) are co-phasor with each other.

iv) All the *E.M.* waves travel in the free space with the same speed (speed of light c) given by:

$$c = \frac{1}{\sqrt{\mu_0\,\varepsilon_0}} = 3 \times 10^8 \text{ m/sec} \qquad (8.49)$$

In a material medium the speed of the *E.M.* waves may be given as:

$$v = \frac{1}{\sqrt{\mu_0\,\mu_r\,\varepsilon_0\,\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\,\varepsilon_r}} = \frac{c}{n} \qquad (8.50)$$

Where, $n \rightarrow \sqrt{\mu_r\,\varepsilon_r}$ (refractive index of the medium / material) *v*) The amplitude ratio of electric field to magnetic field in an *E.M.* wave may be given as:

$$\frac{E}{B} = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad (8.51)$$

vi) The *E.M.* waves carry energies along with them, while they propagate in the space. This energy is shared between the electric field and the magnetic field equally. The average energy density of an *E.M.* wave may be given as:

$$u = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \quad (8.52)$$

vii) The *E.M.* waves transport linear momentum as they propagate in the space given as:

$$p = \frac{U}{c} \quad (8.53)$$

viii) The *E.M.* waves are not deflected by electric fields or by magnetic fields.

ix) The *E.M.* waves obey the principle of superposition. They show the properties of reflection, refraction, interference, diffraction and polarization.

x) The electric field of an *E.M.* wave is responsible for its optical effects as $E \gg B$ for an *E.M.* wave.

Problem 8.11: *An electromagnetic wave travels in a medium at an speed of* 2×10^8 *m/sec. The relative permeability of the medium is* 1. *Determine the relative permittivity of the material.*

Solution: $v = 2 \times 10^8$ m/sec, $\mu_r = 1$

The speed of an electromagnetic wave in a material medium may be given as:

$$v = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \qquad \text{So,} \qquad \varepsilon_r = \frac{1}{\mu_r} \times \left(\frac{c}{v}\right)^2 = \frac{1}{1} \times \left(\frac{3 \times 10^8}{2 \times 10^8}\right)^2 = 2.25$$

Problem 8.12: *A plane electromagnetic wave of frequency* 25 *MHz travels in free space along the x-direction. At a particular point in space and time,* $\vec{E} = 6.3\ \hat{j}$ *V/m. Determine the value of the magnetic field* ($\vec{B}$) *at this point.* [NCERT, CBSE 2005-06]

Solution: $\nu = 25$ MHz, $\vec{E} = 6.3\ \hat{j}$ V/m

The magnitude of the magnetic field may be given as:

$$B = \frac{E}{c} = \frac{6.3}{3 \times 10^8} = 2.1 \times 10^{-8} \text{ T}$$

We know that the direction of propagation of wave may be given as:

$$\text{Direction of propagation} = \vec{E} \times \vec{B} = \hat{i} \qquad \text{or,} \qquad \hat{j} \times \hat{k} = \hat{i}$$

So, magnetic field is oscillating along *z*-axis.

i.e. $\vec{B} = 3.1 \times 10^{-8}\ \hat{k}$ T

Problem 8.13: *The magnetic field in a plane electromagnetic wave propagating along z-axis is given by:*

$$B_y = 2 \times 10^{-7} \sin (0.5 \times 10^3 x + 1.5 \times 10^{11} t) \text{ Tesla}$$

Determine: i) wavelength and frequency of the wave, ii) the expression for the electric field of the electromagnetic wave. [NCERT]

Solution: $B_y = 2 \times 10^{-7} \sin (0.5 \times 10^3 x + 1.5 \times 10^{11} t)\ \hat{j}$ Tesla

Comparing the equation with the standard equation for magnetic field,

$$B = B_0 \sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$$

We get:

$$B_0 = 2 \times 10^{-7} \text{ Tesla}$$

$$\frac{2\pi}{\lambda} = 0.5 \times 10^3 \qquad \text{or,} \qquad \lambda = \frac{2\pi}{0.5\times10^3} = 12.5 \text{ mm}$$

And, $$\frac{2\pi}{T} = 1.5 \times 10^{11} \qquad \text{or,} \qquad \nu = \frac{1}{T} = \frac{1.5\times10^{11}}{2\pi} = 2.387 \times 10^{10} \text{ Hz} = 23.87 \text{ GHz}$$

The amplitude of the electric field may be given as:

$$E_0 = B_0\, c = 2 \times 10^{-7} \times 3 \times 10^8 = 60 \text{ V/m}$$

We know that the direction of propagation of wave may be given as:

$$\text{Direction of propagation} = \vec{E} \times \vec{B} = \hat{k} \qquad \text{or,} \qquad \hat{i} \times \hat{j} = \hat{k}$$

So, the electric field is oscillating along *x*-axis, and the expression for the electric field may be given as:

$$E_x = 60 \sin(0.5 \times 10^3\, x + 1.5 \times 10^{11}\, t)\ \hat{i} \text{ V/m}$$

Problem 8.14: *A light wave with an energy flux of* 18 *watts/cm*^{2} *falls on a non-reflecting surface at normal incidence. If the surface has an area of* 20 *cm*^{2}*, determine the average force exerted on the surface during a* 30 *minute time span.* [NCERT]

Solution: Energy Flux = 18 Watt/cm^2, A = 20 cm^2, t = 30 minute

The total energy incident on the surface during 30 minutes time span:

$$U = \textit{Energy flux} \times A \times t = 18 \times 20 \times 30 \times 60 = 6.48 \times 10^5 \text{ Joules}$$

The equivalent momentum of the energy of wave on a non-reflecting surface may be given as:

$$p = \frac{U}{c} = \frac{6.48\times10^5}{3\times10^8} = 2.16 \times 10^{-3} \text{ kg-m/sec}$$

The force exerted may be given as:

$$F = \frac{p}{t} = \frac{2.16\times10^{-3}}{30\times60} = 1.2 \times 10^{-6} \text{ kg-m/sec}^2 = 1.2 \times 10^{-6} \text{ N}$$

Problem 8.15: *Determine the amplitude of electric field and the magnetic field produced by the radiation coming from a* 100 *W lamp at a distance of* 3 *m. Assume that the efficiency of the lamp is* 2.5% *and it is a point source.* [NCERT]

Solution: P = 100 Watt, l = 3 m, η_{lamp} = 2.5%

The lamp being a point source will radiates light in all directions uniformly (spherically). The area of the sphere at a distance of 3 m from the lamp may be given as:

$$A = 4\pi r^2 = 4\pi \times (3)^2 = 113.01 \text{ m}^2$$

The intensity of the light at 3 m from the lamp may be given as:

$$I = \frac{\text{Power}}{\text{Area}} = \frac{2.5\% \text{ of lamp power}}{A} = \frac{0.025\times100}{113.1} = 0.0221 \text{ Watt/m}^2 = 22.1 \text{ mW/m}^2$$

The intensity of the electromagnetic wave may be given as:

$$I_{e.m.wave} = \varepsilon_0 \, E_{rms}^2 \; c = \frac{1}{\mu_0} \; B_{rms}^2 \; c$$

Half of the intensity of the wave is due to the electric field and half of the intensity is due to magnetic field.

So, $\frac{1}{2} \times \varepsilon_0 \, E_{rms}^2 \; c = \frac{1}{2} \times 0.0221$

or, $E_{rms} = \sqrt{\frac{0.0221}{\varepsilon_0 \, c}} = \sqrt{\frac{0.0221}{8.85\times10^{-12}\times3\times10^8}} = 2.885 \text{ V/m}$

So, $E_0 = \sqrt{2} \times E_{rms} = \sqrt{2} \times 2.885 = 4.08 \text{ V/m}$

and, $\frac{1}{2\mu_0} \; B_{rms}^2 \; c = \frac{1}{2} \times 0.0221$

or, $B_{rms} = \sqrt{\frac{0.0221\times\mu_0}{c}} = \sqrt{\frac{0.0221\times4\pi\times10^{-7}}{3\times10^8}} = 9.62 \times 10^{-9} \text{ T}$

So, $B_0 = \sqrt{2} \times B_{rms} = \sqrt{2} \times 9.62 \times 10^{-9} = 1.36 \times 10^{-8} \text{ T}$

Problem 8.16: *A plane electromagnetic wave in the visible region is moving along the z-direction. The frequency of the wave is* 5×10^{14} *Hz and the electric field at any point is varying in sinusoidal manner (with time) having an amplitude of* 1 *V/m. Determine the average values of the energy densities of the electric field and magnetic field.*

Solution: $\nu = 5 \times 10^{14}$ Hz, $E_0 = 1$ V/m

The energy density of the electric field may be given as:

$$u_E = \frac{1}{4} \; \varepsilon_0 \; E_0^2 = \frac{1}{4} \times 8.85 \times 10^{-12} \times (1)^2 = 2.213 \times 10^{-12} \text{ J/m}^3$$

The energy density of the magnetic field may be given as:

$$u_B = \frac{1}{4\mu_0} \; B_0^2 = \frac{1}{4\mu_0} \times \left(\frac{E_0}{c}\right)^2 = \frac{1}{4\times4\pi\times10^{-7}} \times \left(\frac{1}{3\times10^8}\right)^2 = 2.21 \times 10^{-12} \text{ J/m}^3$$

Problem 8.17: *The electric field of an electromagnetic wave may be given as:*

$$E = 50 \, sin \, \frac{2\pi}{\lambda} \, (c\,t - x) \; V/m$$

Determine the energy contained in a cylinder of cross section 10 *cm*^{2} *and length* 50 *cm along the x-axis.*

Solution: $E = 50 \sin \frac{2\pi}{\lambda} (c\,t - x)$ V/m, $A = 10$ cm^2, $l = 50$ cm

Average energy density of the electromagnetic wave may be given as:

$$u_{e.m.} = \frac{1}{2}\ \varepsilon_0\ E_0^2\ = \frac{1}{2}\ \times 8.85 \times 10^{-12} \times (50)^2 = 1.10625 \times 10^{-8}\ \text{J/m}^3$$

The energy contained in the required cylinder may be given as:

$$U = u_{e.m.} \times A \times l = 1.10625 \times 10^{-8} \times 10 \times 10^{-4} \times 0.50 = 5.53125 \times 10^{-12}\ \text{J}$$

Problem 8.18: *A plane electromagnetic wave propagating in the x-direction has a wavelength of* 6 *mm. The electric field is in the y-direction and its amplitude is* 30 *V/m. Write down the suitable equation for the electric field and the magnetic field as a function of x and t.*

Solution: $\lambda = 6$ mm, $E_0 = 30$ V/m

The time period of the electromagnetic wave may be given as:

$$T = \frac{1}{\nu} = \frac{\lambda}{c} = \frac{6\times10^{-3}}{3\times10^{8}} = 2 \times 10^{-11}\ \text{sec}$$

The equation for the electric field may be given as:

$$E = E_y = E_0 \sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right) = 30 \sin\left(\frac{2\pi}{6\times10^{-3}}\times x - \frac{2\pi}{2\times10^{-11}}\times t\right)$$

or, $E = 30 \sin (1047.2\ x - 3.14 \times 10^{11}\ t)$ V/m

The equation for the magnetic field may be given as:

$$B = B_z = B_0 \sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right) = \frac{E_0}{c} \sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$$

$$= \frac{30}{3\times10^{8}} \sin\left(\frac{2\pi}{6\times10^{-3}}\times x - \frac{2\pi}{2\times10^{-11}}\times t\right)$$

or, $B = 1 \times 10^{-7} \sin (1047.2\ x - 3.14 \times 10^{11}\ t)$ Tesla

Problem 8.19: *A laser beam has intensity of* 2.5×10^{14} *W/m*^{2}*. Determine the amplitude of electric field and the magnetic field in the beam.*

Solution: $I = 2.5 \times 10^{14}\ \text{W/m}^2$

We know that for an electromagnetic wave:

$$I = u_{avg} \times c = \frac{1}{2}\ \varepsilon_0\ E_0^2\ c$$

or, $$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2\times 2.5\times10^{14}}{8.85\times10^{-12}\times3\times10^{8}}} = 4.34 \times 10^{8}\ \text{V/m}$$

So, the amplitude of the magnetic field may be given as:

$$B_0 = \frac{E_0}{c} = \frac{4.34\times10^{8}}{3\times10^{8}} = 1.447\ \text{T}$$

Problem 8.20: *A light beam is travelling in the x-direction whose electric field may be given as:*

$$E_y = 270 \sin \omega\left(t - \frac{x}{c}\right)\ \text{V/m}$$

An electron is constrained to move along the y-direction with a speed of 2×10^{7} *m/sec. Determine the maximum electric force and maximum magnetic force on the electron.*

Solution: $E_y = 270 \sin \omega\left(t - \frac{x}{c}\right)$ V/m, $v_e = 2 \times 10^{7}$ m/sec

Comparing the expression for the electric field with the standard equation, we get:

$E_0 = 270$ V/m

The amplitude of the magnetic field may be given as:

$$B_0 = \frac{E_0}{c} = \frac{270}{3\times10^8} = 9 \times 10^{-7} \text{ T}$$

The maximum force experienced by the electron due to the electric field may be given as:

$$F_e = e\,E_0 = 1.6 \times 10^{-19} \times 270 = 4.32 \times 10^{-17} \text{ N}$$

The maximum force experienced by the electron due to the magnetic field may be given as:

$$F_B = e\,v_e\,B = 1.6 \times 10^{-19} \times 2 \times 10^7 \times 9 \times 10^{-7} = 2.88 \times 10^{-18} \text{ N}$$

Problem 8.21: *The electric field vector of a plane electromagnetic wave oscillates in sinusoidal manner at a frequency of* 4.5×10^{10} *Hz. Determine the wavelength of this electromagnetic wave.*
[CBSE 1990-91]

Solution: $\nu = 4.5 \times 10^{10}$ Hz

The wavelength of the electromagnetic wave may be given as:

$$\lambda = \frac{c}{\nu} = \frac{3\times10^8}{4.5\times10^{10}} = 6.667 \text{ mm}$$

Problem 8.22: *The amplitude of the electric field in a plane electromagnetic wave is* 600 *V/m. The wave is propagating along the x-direction and the electric field is along the y-direction. Determine the amplitude and direction of the magnetic field of the electromagnetic wave.*

Solution: $(E_0)_y = 600$ V/m

The amplitude of the magnetic field of the electromagnetic wave may be given as:

$$B_0 = \frac{E_0}{c} = \frac{600}{3\times10^8} = 2 \times 10^{-6} \text{ T}$$

The electric field is along the $\hat{j}$ direction and the electromagnetic wave is propagating along the $\hat{i}$ direction, we know that the direction of propagation of the electromagnetic wave may be given as:

Direction of Propagation = $\vec{E} \times \vec{B} = \hat{i}$ or, $\hat{j} \times \hat{k} = \hat{i}$

So, the direction of magnetic field is along the z-axis, and the magnetic field may be given as:

$$(B_0)_z = 2 \times 10^{-6}\ \hat{k} \text{ T}$$

Problem 8.23: *The frequencies in the AM broadcast band range from* 0.55 *MHz to* 1.6 *MHz. Determine the longest and the shortest wavelength in this band.*

Solution: $\nu_1 = 0.55$ MHz, $\nu_2 = 1.6$ MHz

The longest wavelength and the shortest wavelength in this band may respectively be given as:

$$\lambda_{longest} = \frac{c}{\nu_1} = \frac{3\times10^8}{0.55\times10^6} = 545.45 \text{ m}$$

and, $$\lambda_{shortest} = \frac{c}{\nu_2} = \frac{3\times10^8}{1.6\times10^6} = 187.5 \text{ m}$$

Problem 8.24: *A radio transmitter operates at a frequency of* 880 *kHz and a power of* 10 *kW. Determine the number of photons emitted per second.* [CBSE 1989-90]

Solution: $\nu = 880$ kHz, $P = 10$ kW

Since the energy of a photon may be given as $E = h\nu$, so the number of photons emitted per second may be given as:

$$n = \frac{\text{Total energy per second}}{h\nu} = \frac{P}{h\nu} = \frac{10\times10^{3}}{6.6\times10^{-34}\times880\times10^{3}} = 1.72\times10^{31}\text{ photons/sec}$$

Problem 8.25: *The permittivity and permeability of the free space are given as:* $\varepsilon_0 = 8.85\times10^{-12}\ C^2\ N^{-1}\ m^{-2}$ *and* $\mu_0 = 4\,\pi\times10^{-7}\ T\ m\ A^{-1}$ *respectively. Determine the speed of the electromagnetic wave in the free space.*

Solution: $\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$, $\mu_0 = 4\,\pi\times10^{-7}\ \text{T m A}^{-1}$

The speed of the electromagnetic wave in the free space may be given as:

$$c = \frac{1}{\sqrt{\mu_0\,\varepsilon_0}} = \frac{1}{\sqrt{4\pi\times10^{-7}\times8.85\times10^{-12}}} = 3\times10^{8}\text{ m/sec}$$

Problem 8.26: *The amplitude of the magnetic field is* 5×10^{-6} *Tesla, in a plane electromagnetic wave with frequency* 1×10^{12} *Hz. Determine the amplitude of the electric field and the average energy density of this electromagnetic wave.*

Solution: $B_0 = 5\times10^{-6}$ T, $\nu = 1\times10^{12}$ Hz

The amplitude of the electric field of the electromagnetic wave may be given as:

$$E_0 = B_0\,c = 5\times10^{-6}\times3\times10^{8} = 1500\text{ V/m}$$

The average energy density of the electromagnetic wave may be given as:

$$u = \frac{1}{2}\,\varepsilon_0\,E_0^2 = \frac{1}{2}\times8.85\times10^{-12}\times(1500)^2 = 9.56\times10^{-6}\text{ J/m}^3$$

Problem 8.27: *A plane electromagnetic wave, with a frequency of* 10^{15} *Hz and a sinusoidally varying electric field of amplitude 2 V/m, is moving along x-direction. Determine the average densities of electric and magnetic fields.*

Solution: $\nu = 10^{15}$ Hz, $E_0 = 2$ V/m

The energy density of the electric field and magnetic field may respectively be given as:

$$u_E = \frac{1}{4}\,\varepsilon_0\,E_0^2 = \frac{1}{4}\times8.85\times10^{-12}\times(2)^2 = 8.85\times10^{-12}\text{ J/m}^3$$

and, $$u_B = \frac{1}{4\mu_0}\,B_0^2 = \frac{1}{4\mu_0}\times\left(\frac{E_0}{c}\right)^2 = \frac{1}{4\times4\pi\times10^{-7}}\times\left(\frac{2}{3\times10^{8}}\right)^2 = 8.85\times10^{-12}\text{ J/m}^3$$

Problem 8.28: *The magnetic field of a plane electromagnetic wave is given as:*

$$B_0 = (200\ \mu T)\ sin\ (4\times10^{-5}\ sec^{-1})\times\left(t-\frac{x}{c}\right)$$

Determine the amplitude of electric field and the average energy density corresponding to it.

Solution: $B = (200\ \mu\text{T})\ \sin\ (4\times10^{-5}\ \text{rad/sec}^{-1})\times\left(t-\frac{x}{c}\right)$

Comparing the equation of magnetic field with the standard expression for the magnetic field:

$$B = B_0 \sin \omega \left(t - \frac{x}{c}\right), \quad \text{We get:} \quad B_0 = 200\ \mu\text{T}, \qquad \omega = 4 \times 10^{-5}\ \text{rad/sec}$$

The amplitude of the electric field may be given as:

$$E_0 = B_0\, c = 200 \times 10^{-6} \times 3 \times 10^{8} = 60\ \text{kV/m}$$

The average energy density corresponding to the electric field may be given as:

$$u_E = \frac{1}{4}\ \varepsilon_0\ E_0^2 = \frac{1}{4} \times 8.85 \times 10^{-12} \times (60 \times 10^3)^2 = 7.965 \times 10^{-3}\ \text{J/m}^3$$

Problem 8.29: *A millimeter wave has a wavelength of* 2 *mm and the oscillating electric field associated with it has an amplitude of* 20 *V/m. Determine the frequency of oscillations of the electric and magnetic fields of this electromagnetic wave. Also, determine the amplitude of magnetic field oscillations.*

Solution: $\lambda = 2$ mm, $E_0 = 20$ V/m

The frequency of oscillations of electric field and magnetic field may be given as:

$$\nu = \frac{c}{\lambda} = \frac{3\times10^8}{2\times10^{-3}} = 1.5 \times 10^{11}\ \text{Hz}$$

The amplitude of the magnetic field oscillations may be given as:

$$B_0 = \frac{E_0}{c} = \frac{20}{3\times10^8} = 6.667 \times 10^{-8}\ \text{T}$$

Problem 8.30: *The electric field varies with an amplitude of* 1 *V/m, in a plane electromagnetic wave having a frequency of* 5×10^{14} *Hz. The wave is propagating along z-axis. Determine the average energy density of: i*) *electric field, ii*) *magnetic field, iii*) *electromagnetic wave, iv*) *amplitude of magnetic field.*

Solution: $E_0 = 1$ V/m, $\nu = 5 \times 10^{14}$ Hz

The energy density of the electric field and magnetic field may respectively be given as:

$$u_E = \frac{1}{4}\ \varepsilon_0\ E_0^2 = \frac{1}{4} \times 8.85 \times 10^{-12} \times (1)^2 = 2.2125 \times 10^{-12}\ \text{J/m}^3$$

$$u_B = \frac{1}{4\mu_0}\ B_0^2 = \frac{1}{4\mu_0} \times \left(\frac{E_0}{c}\right)^2 = \frac{1}{4\times4\pi\times10^{-7}} \times \left(\frac{1}{3\times10^8}\right)^2 = 2.21 \times 10^{-12}\ \text{J/m}^3$$

The energy density of the electromagnetic wave may be given as:

$$u = u_E + u_B = 2.2125 \times 10^{-12} + 2.21 \times 10^{-12} = 4.4225 \times 10^{-12}\ \text{J/m}^3$$

$$= \frac{1}{2}\ \varepsilon_0\ E_0^2 = \frac{1}{2} \times 8.85 \times 10^{-12} \times (1)^2 = 4.425 \times 10^{-12}\ \text{J/m}^3$$

The amplitude of the magnetic field oscillations may be given as:

$$B_0 = \frac{E_0}{c} = \frac{1}{3\times10^8} = 3.333 \times 10^{-9}\ \text{T}$$

8.15 Electromagnetic Spectrum: All the known electromagnetic radiations from a very big family of electromagnetic waves stretch over a wide range of frequency and wavelength.

The orderly arrangement of all the known electromagnetic waves in accordance with their wavelength / frequency into various groups, having peculiar properties, is known as electromagnetic spectrum.

The complete electromagnetic spectrum is shown in the Fig. 8.10. The main parts of the electromagnetic spectrum are Gamma (γ) rays, *X*-rays, ultraviolet rays, visible light, infrared rays, microwaves and radio waves in the order of increasing wavelength from 10^{-2} Å (10^{-12} m) to 10^{6} m.

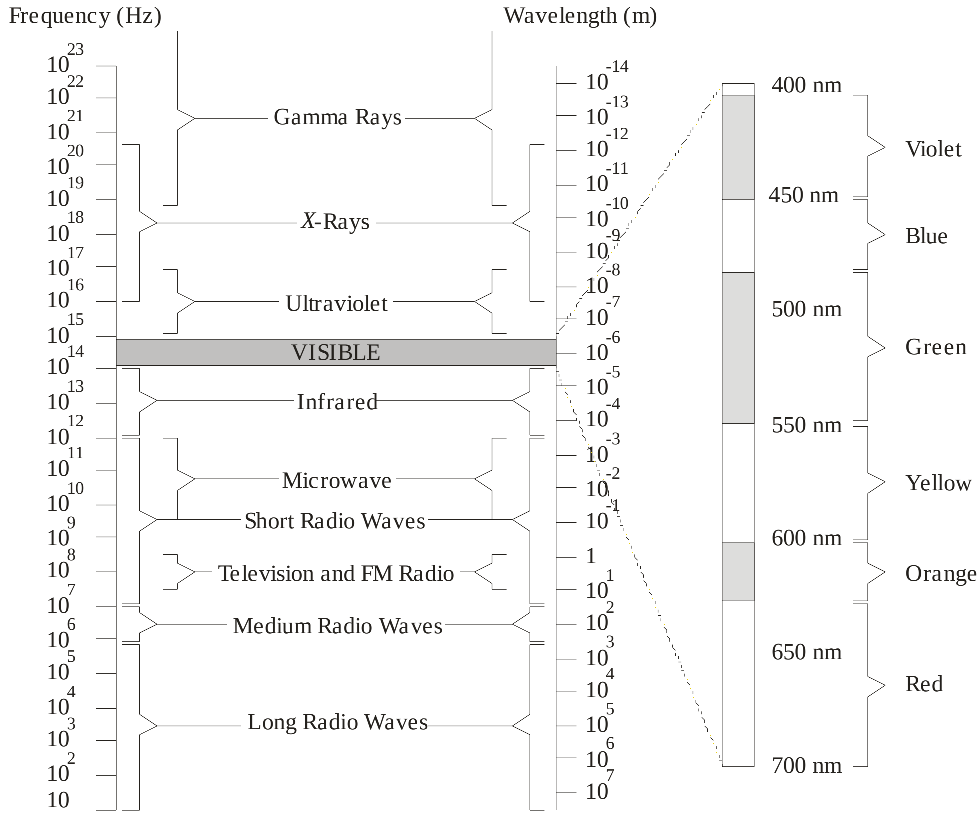


Fig. 8.10

The various regions of electromagnetic spectrum do not have sharply defined boundaries and they may overlap each other. The classification is based on their basic properties and how the waves are produced and / or detected.

An analysis of the electromagnetic spectrum in the order of increasing frequency is given below.

***i*)** **Radio Waves:** They are the electromagnetic waves of longest wavelength (known) and the minimum frequency.

1.	Wavelength Range	600 m – 0.1 m
2.	Frequency Range	500 kHz – 1000 MHz
3.	Source	Accelerated motion (oscillation) of charges in conducting wires.
4.	Discovered by	Marconi in 1895
5.	Properties	Reflection, diffraction.

Uses of Radio Waves: The uses of radio waves are given below:

i) They are used in radio and television communication systems.

ii) They are used in radio-astronomy.

Some important Wire-less Communication Bands are given below:

Sr. No.	Frequency Band	Service
1	540 Hz – 1600 kHz	Medium Wave AM Radio Band
2	3 – 30 MHz	Short wave AM Radio Band
3	88 – 108 MHz	FM Radio Broadcast
4	54 – 890 MHz	Television Broadcast
5	840 – 935 MHz	Cellular Mobile Operator's Band

***ii*) Micro Waves:** They are the electromagnetic waves having wavelengths next smaller to radio waves.

1.	Wavelength Range	0.3 m – 10^{-3} m
2.	Frequency Range	10^{9} Hz – 10^{12} Hz
3.	Source	Oscillating currents in special vacuum tubes like klystrons, magnetrons and Gunn diodes.
4.	Discovered by	Marconi in 1895
5.	Properties	Reflection, refraction, diffraction and polarization, they can travel as a beam of signal due to their shorter wavelength and higher energy levels.

Uses of Micro Waves: The uses of micro waves are given below:

i) They are used in radar systems for aircraft navigation.

ii) They are used in long distance communication system via geostationary satellites.

iii) They are used in microwave ovens.

iii*) Infrared Waves (*IR Radiations*):** They are the electromagnetic radiations lying close to the low frequency or long wavelength visible spectrum. Infrared waves produce heating effect, so they are also known as ***heat waves or ***thermal radiations***. The water (H_2O) molecules, CO_2 molecules and NH_3 molecules present in different materials readily absorb infrared waves, and increases their thermal motions and heats up the material and their surroundings.

1.	Wavelength Range	5×10^{-3} m – 10^{-6} m
2.	Frequency Range	10^{11} Hz – 5×10^{14} Hz
3.	Source	Hot bodies and molecules.
4.	Discovered by	William Herschel in 1800.
5.	Properties	Heating effect, reflection, refraction, diffraction and propagation through fog.

Uses of Infrared Waves: The uses of infrared waves are given below:

i) They are used in remote controls, which have a small infrared transmitter.

ii) They are used in green houses to keep the plants warm.

iii) They are used in haze photography, because infrared waves are less scattered than visible light due to atmospheric particles.

iv) They are used in infrared lamps in the treatment of muscular problems.

v) They are used in the study of molecular structure of substances and alloys.

***iv*) Visible Light:** *It is a very small part of the electromagnetic spectrum towards which the human retina is sensitive*. The visible light emitted or reflected from bodies around us gives information about the world through the vision.

1.	Wavelength Range	8×10^{-7} m – 4×10^{-7} m (8000 Å – 4000 Å)
2.	Frequency Range	4×10^{14} Hz – 7×10^{14} Hz
3.	Source	Radiated by excited atoms in ionized gases and incandescent bodies and sun.
4.	Properties	Reflection, refraction, interference, diffraction, polarization, photo electric effect, photographic action, sensation of sight.

Uses of Visible Light: The uses of visible light are given below:

i) The visible light emitted or reflected from bodies around us gives information about the world through the vision.

ii) The visible light can cause or accelerate various chemical reactions.

***v*) Ultraviolet Light (*UV Rays*):** They are the electromagnetic waves having wavelengths just smaller than visible light and can be detected just beyond the violet end of the solar spectrum.

1.	Wavelength Range	3.5×10^{-7} m – 1.5×10^{-7} m
2.	Frequency Range	10^{16} Hz –10^{17} Hz
3.	Source	High voltage gas discharge tubes, arcs of iron and mercury and sun.
4.	Discovered by	Ritter in 1800.
5.	Properties	Effect on photographic plates, fluorescence, ionization, highly energetic, tanning of human skin.

Uses of Ultraviolet Waves: The uses of ultraviolet waves are given below:

i) They are used in food preservation.

ii) They are used in the study of invisible writing, forged documents and finger prints.

iii) They are used in the study of molecular structure of substances and alloys.

The ultraviolet radiations in large quantities have harmful effects on human beings. But fortunately, most of the ultraviolet light coming from the sun is absorbed by the *ozone layer* in the atmosphere at altitude of about 40-50 km.

***vi*) *X*-Rays:** They are the electromagnetic waves having wavelengths just smaller than ultraviolet rays. The *X*-rays can pass through many forms of matter, so they have many useful medical and industrial applications.

1.	Wavelength Range	100 Å – 0.1 Å
2.	Frequency Range	10^{18} Hz –10^{20} Hz
3.	Source	Sudden retardation of fast moving electrons by a metal target.
4.	Discovered by	Rontgen in 1895.
5.	Properties	Effect on photographic plates, ionization of gases, photo electric effect, fluorescence, more energetic than Ultraviolet rays.

Uses of *X*-Rays: The uses of *X*-rays are given below:

i) They are used in medical diagnosis, because *X*-rays can pass through human flesh but not through the bones.

ii) They are used in study of crystal structures because *X*-rays can be reflected and diffracted by crystals.

iii) They are used in *engineering* for detecting faults, cracks, flaws and holes or air bubbles trapped in finished metal products.

iv) They are used in *detective departments* for searching explosives, diamond, gold jewelry *etc*. in the possession of smugglers.

v) They are used in radio-therapy to cure untraceable skin diseases and malignant growths.

***vii*) Gamma Rays:** They are the electromagnetic waves having highest known frequency range and lowest wavelength range. They are the most penetrating electromagnetic waves and are very-very dangerous and harmful for human beings.

1.	Wavelength Range	10^{-14} m – 10^{-10} m
2.	Frequency Range	10^{18} Hz –10^{22} Hz
3.	Source	Radioactive nuclei and nuclear reactions. Co – 60 is a pure γ-ray source.
4.	Discovered by	Henry Becquerel in 1896.
5.	Properties	Effect on photographic plates, fluorescence, ionization, diffraction, highest penetrating power.

Uses of Gamma Rays: The uses of γ-rays are given below:

i) They are used in radio-therapy for the treatment of malignant tumors.

ii) They are used in the manufacture of polyethylene from ethylene.

iii) They are used to initiate nuclear reactions.

iv) They are used for preserving food stuffs for a long duration, because soft γ-rays can kill organisms.

v) They are used in sterilization (to kill the bio-burden) of the products to be used in operation theaters.

vi) They are used in study of structure of atomic nuclei.

8.16 Earth's Atmosphere: *The thick envelope of air surrounding the earth is known as earth's atmosphere.* The earth's atmosphere extends to about 400 km above the earth's surface. The air pressure goes on gradually decreasing as soon as we go up in the earth's atmosphere. The earth's atmosphere may broadly be divided into the following layers / zones.

i*) Troposphere:** This layer of atmosphere extends above sea level up to a height of 12 km. Its upper boundary is known as ***tropopause. The temperature decreases from 290 K to 220 K with increase in height at the *tropopause*. Troposphere contains a large amount of water vapors and cloud formation takes place in this layer only. This layer is important from the point of view of all type of weather phenomenon that affects our environment.

ii*) Stratosphere:** This layer extends from 12 km to 50 km above sea level. Its upper boundary is known as ***stratopause. The lower portion of this layer contains a large concentration of ozone, resulting from the dissociation of molecular oxygen by solar ultraviolet radiation in the upper atmosphere. The layer containing ozone is known as ***Ozone Layer*** or ***Ozonosphere***, and it extends from 15 km to about 30 km above the sea level. The temperature of stratosphere rises from 220 K to 280 K.

iii*) Mesosphere:** This layer extends from 50 km to 80 km above the sea level. Its upper boundary is known as ***mesopause. The temperature of this region falls from 280 K to 180 K.

iv*) Ionosphere:** This layer extends from 80 km to about 400 km above sea level. The temperature in this layer increases with height from 180 K to 700 K. The ionosphere is mostly composed of electrons and positive ions. The ionization of gases in this layer is caused by the ultraviolet radiations and *X*-rays coming from the sun. The lower portion of the ionosphere extending from 80 km to 95 km is called ***thermosphere. The concentration of electrons (*i.e.* electron density) is found to be very large in a region beyond 110 km from the sea level, which extends vertically for a few kilometers. This layer of electrons is called ***Kennelly Heaviside Layer***. The electron density decreases beyond this layer considerably up to a height of 250 km. An electron rich layer is again met beyond 250 km and is known as ***Appleton Layer***.

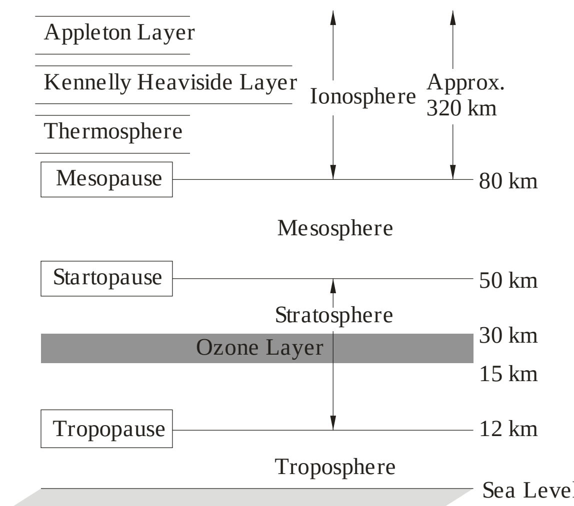


Fig. 8.11

All the layers of the earth's atmosphere are composed of neutral molecules except Ionosphere, where charged particles and ions are present in abundance. The relative permittivity of the ionosphere is less than one (ε_0 = 1 *for air*), *so the electromagnetic rays may bend away from the normal while they travels from earth's surface into the outer space, going from the denser medium to rarer medium*.

8.17 Effects of Earth's Atmosphere on Electromagnetic Radiations: The sun is the main source of the electromagnetic radiations, that we receive on the earth. The atmosphere is transparent to the visible radiations, as we can see the sun and the stars through it clearly most of the times. However, the other components such as infrared and ultraviolet radiations from the sun are absorbed in different layers of the atmosphere.

Green House Effect of Earth's Atmosphere: The radiations from the sun heats up the earth's surface, due to which earth heats up to a moderate temperature (20°C to 50°C). The earth re-radiates the heat absorbed mostly in the infrared region, due to its lower temperature (20°C to 50°C).

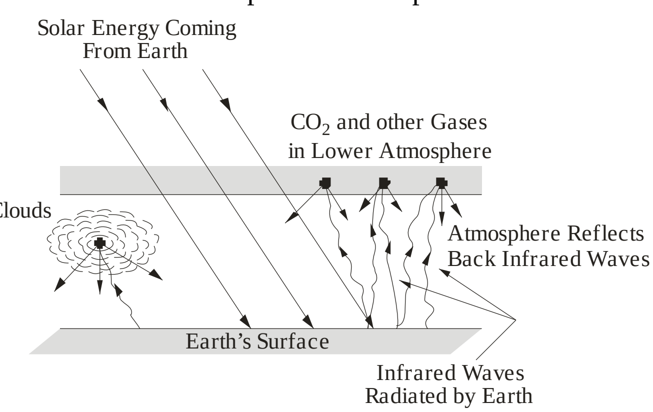


Fig. 8.12

The infrared radiations from the earth get reflect back towards the earth's surface from the CO_2 and other gases in the lower atmosphere and from the clouds, as shown in the Fig. 8.12. So, the heat is entrapped inside the atmosphere of the earth to keep its atmosphere warm even in the absence of the sun during night. So, "*The effect of the atmosphere of the earth, which maintains the moderate temperature of the earth even in absence of the sun, is known as green house effect*". That is the reason due to which ***a cloudy night appears to be warmer than that of any other night when sky is clear from the clouds***.

Importance of Ozone Layer: The solar radiations also has ultraviolet radiations and some other lower wavelength radiations as its constituents, which may cause genetic damage to living cells and so may be very dangerous and harmful for human beings and other living non-living creatures on the earth. The ozone layer is formed due to these radiations only, and it absorbs these radiations from the sun and prevents them from reaching the earth's surface and causing damage to life on the earth. It also helps the lower atmosphere to create the green house effect on the earth.

SHORT ANSWER TYPE QUESTIONS FOR EXERCISE

1. Mention the *inconsistency in the Ampere's Circuital Law* pointed out by the *Maxwell*, with the help of a suitable example and diagrams.
2. Explain the concept of *Maxwell's displacement current*, and hence the *Maxwell's modification of Ampere's Circuital Law.*
3. Explain the *consistency of the Modified Ampere's Circuital Law*, and hence the *continuity of current* in a closed circuit having a capacitor in series.
4. Give the important properties of the *displacement current*, and give the *Maxwell's equations* for the *basic principle of electromagnetism.*
5. Explain the reasons which enabled *Maxwell* to predict the existence of electromagnetic waves. Also, give the *mathematical representation* of an *electromagnetic wave*.
6. Derive the *value of speed* of *electromagnetic waves* from *Maxwell's electromagnetic waves predication*, and hence show that *light is an electromagnetic wave* and has *transverse nature*.
7. Explain *Hertz experiment for generation and prediction of electromagnetic waves*.
8. Summarize the *history of prediction and observation of electromagnetic waves*.
9. Define and give a relevant expression for following parameters of electromagnetic waves: *i*) *energy density, ii*) *intensity, iii*) *momentum, iv*) *pressure exerted.*
10. Explain the *important properties of electromagnetic waves*.
11. Write down a short note about *properties* and *applications* of the following *electromagnetic waves*: *i*) *radio waves, ii*) *microwaves, iii*) *IR radiations, iv*) *visible light, v*) *UV rays, vi*) *X-rays, vii*) γ*-rays.*
12. How many *layers* are there in the *atmosphere of earth*? Explain briefly about them.
13. Explain: *i*) *green-house effect, ii*) *importance and cause of ozone layer.*